

Conditional teleportation using optical squeezers and photon counting

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We suggest a scheme of conditional teleportation of quantum states of optical fields using squeezers and photon counting. Alice feeds the mode whose state is desired to be teleported and one mode of a two-mode squeezed vacuum into a parametric amplifier and detects output photon numbers. The result is then communicated to Bob who shifts the photon number of his part accordingly. We show that for some classes of states the method can yield, with reasonable success probability, a teleportation fidelity close to unity. The method is a principally realizable modification of a recently proposed scheme [G.J. Milburn and S.L. Braunstein, Phys. Rev. A **60**, 937 (1999)], where measurements of the photon-number difference and the phase sum are considered.

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I. INTRODUCTION

In quantum teleportation, an unknown state of a system is destroyed and created on another, distant system of the same type. The method was first suggested in [1] and realized in [2] for discrete variables, namely photonic (polarization) qubits. Subsequently, the concept has been extended to continuous variables [3,4], and then realized experimentally to teleport a coherent state by means of parametrically entangled (squeezed) optical beams and quadrature-component measurements [5]. The concept of teleportation of continuous quantum variables has been further elaborated in [6].

The basic requirement of quantum teleportation is that the two parties share an entangled state with each other. In continuous-variable teleportation of quantum states of optical field modes, a two-mode squeezed vacuum is suited for playing the role of the entangled state. The quadrature components \hat{q}_k and \hat{p}_k ($[\hat{q}_k, \hat{p}_k] = i$, $k = 1, 2$) are correlated and anti-correlated, respectively, such that $\Delta(\hat{q}_1 - \hat{q}_2) < 1$ and $\Delta(\hat{p}_1 + \hat{p}_2) < 1$. For large squeezing, the correlations approach the original Einstein-Podolsky-Rosen (EPR) correlations [7] (for EPR correlations in optical fields, see, e.g., [8,9]).

The first scheme of teleportation that uses an optical two-mode squeezed vacuum is based on (single-event) quadrature-component measurements exploiting the above mentioned quadrature-component correlations [4]. Later on, it has been realized that there are photon-number and phase correlations in a two-mode squeezed

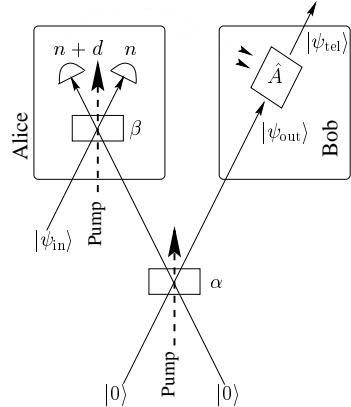


FIG. 1. Teleportation scheme as explained in the first paragraph of Sec. II.

vacuum which could also be used for a potential teleportation protocol [10]. In the scheme in [10] it is assumed that a measurement of the photon-number difference and the phase sum of the two modes on Alice's side is performed. The obtained information is then sent to Bob who has to transform the quantum state of his mode by appropriate phase and photon-number shifting, thus creating the resulting teleported state. The scheme is conditional, as for some measured photon-number differences the state that is desired to be teleported cannot be re-created by Bob.

Unfortunately, the scheme in [10] requires phase measurements for which no methods have been known so far. Is there any hope to realize teleportation based on such a scheme or a related one? In this paper we suggest a viable modification of the scheme proposed in [10] which is based on (single-event) photon-number measurements on the output of a parametric amplifier (squeezer). The scheme is also conditional, and it applies to certain classes of quantum states. Even though the scheme is not universal, it can produce for some states and some measurement events higher teleportation fidelities than the scheme based on quadrature-component measurements [4].

The paper is organized as follows. In Sec. II we present the scheme and derive the expression for the teleported quantum state. In Sec. III we illustrate the method presenting numerical results, and we conclude in Sec. IV.

II. THEORY

Let us consider the scheme sketched in Fig. 1. The entangled state is a two-mode squeezed vacuum produced by the first parametric amplifier from the vacuum state, α being the squeezing parameter. One of the two output modes of the first parametric amplifier is then used as one of the input modes of the second parametric amplifier (squeezing parameter β), and the mode whose quantum state $|\psi_{\text{in}}\rangle$ is desired to be teleported is the other input mode. Alice measures the photon numbers n and $n+d$ ($d \geq -n$) at the output of the second parametric amplifier and communicates the result to Bob. Owing to Alice's measurement, the state of the mode that was sent to Bob from the first parametric amplifier has been projected onto the state $|\psi_{\text{out}}\rangle$. Bob now reproduces the input state by means of the transformation $\hat{A}|\psi_{\text{out}}\rangle = |\psi_{\text{tel}}\rangle$, where the operator \hat{A} shifts the photon number according to the measured photon-number difference d .

The three modes are initially (i.e., before they enter any of the parametric amplifiers) prepared in the states $|\psi_{\text{in}}\rangle$, $|0\rangle$, and $|0\rangle$, where the state $|\psi_{\text{in}}\rangle$ that is desired to be teleported can be written in the Fock basis as

$$|\psi_{\text{in}}\rangle = \sum_k |k\rangle \langle k| \psi_{\text{in}}. \quad (1)$$

After passing the parametric amplifiers and detecting n and n' photons in the outgoing modes (modes 0 and 1) on Alice's side, the state of Bob's mode (mode 2) is

$$|\psi_{\text{out}}\rangle_2 = P^{-\frac{1}{2}} \langle n|_1 \langle n' | \hat{S}_{01}(\beta) \hat{S}_{12}(\alpha) |\psi_{\text{in}}\rangle_0 |0\rangle_1 |0\rangle_2, \quad (2)$$

where P is the probability of that measurement event. The two-mode squeeze operator $\hat{S}_{kl}(\alpha)$ is given by

$$\hat{S}_{kl}(\alpha) = \exp\left(\alpha^* \hat{a}_k \hat{a}_l - \alpha \hat{a}_k^\dagger \hat{a}_l^\dagger\right), \quad (3)$$

with \hat{a}_k (\hat{a}_k^\dagger) being the photon destruction (creation) operator of the k th mode. It can be written in the Fock basis as

$$\begin{aligned} {}_k \langle m|_l \langle m' | \hat{S}_{kl}(\alpha) | n\rangle_k | n'\rangle_l &= \delta_{m-m', n-n'} e^{i(m'-n')\varphi_\alpha} \\ &\times (-1)^{n'} \sqrt{m! m'! n! n'!} \frac{(\sinh |\alpha|)^{n'} (\tanh |\alpha|)^{m'}}{(\cosh |\alpha|)^{n+1}} \\ &\times \sum_{j=\max\{0, n'-n\}}^{\min\{m', n'\}} \frac{(-\sinh^2 |\alpha|)^{-j}}{j!(m'-j)!(n'-j)!(n-n'+j)!}, \end{aligned} \quad (4)$$

where $\alpha = |\alpha| e^{i\varphi_\alpha}$. For the following it is useful to introduce the coefficients

$$\begin{aligned} S_{m'}^m(d; \alpha) &= {}_k \langle m+d|_l \langle m | \hat{S}_{kl}(\alpha) | m'+d\rangle_k | m'\rangle_l \\ &= {}_k \langle m|_l \langle m+d | \hat{S}_{kl}(\alpha) | m'\rangle_k | m'+d\rangle_l \\ &= e^{i(m-m')\varphi_\alpha} (-1)^{m'} \sqrt{m! m'! (m+d)! (m'+d)!} \\ &\times \frac{(\tanh |\alpha|)^{m+m'}}{(\cosh |\alpha|)^{d+1}} \sum_{j=0}^{\min\{m, m'\}} \frac{(-\sinh^2 |\alpha|)^{-j}}{j!(m-j)!(m'-j)!(d+j)!}. \end{aligned} \quad (5)$$

The properties of the conditional quantum state $|\psi_{\text{out}}\rangle$, Eq. (2), in which the mode 2 is prepared after the detection of n and n' photons in the modes 0 and 1 respectively, are qualitatively different for different sign of the observed difference $d = n' - n$. In the case when $d \leq 0$ is valid, then from Eq. (2) together with Eq. (5) it follows that $(|\psi_{\text{out}}\rangle_2 \rightarrow |\psi_{\text{out}}\rangle)$

$$\langle m| \psi_{\text{out}}\rangle = P^{-\frac{1}{2}} S_m^{n+d}(-d; \beta) S_0^m(0; \alpha) \langle m-d| \psi_{\text{in}}\rangle, \quad (6)$$

where the detection probability P is given by

$$P = \sum_m |S_m^{n+d}(-d; \beta)|^2 |S_0^m(0; \alpha)|^2 |\langle m-d| \psi_{\text{in}}\rangle|^2. \quad (7)$$

In the second case when $d > 0$ is valid, we derive

$$\langle m| \psi_{\text{out}}\rangle = \begin{cases} P^{-\frac{1}{2}} S_{m-d}^n(d; \beta) S_0^m(0; \alpha) \langle m-d| \psi_{\text{in}}\rangle, & m \geq d, \\ 0, & m < d, \end{cases} \quad (8)$$

where

$$P = \sum_{m \geq d} |S_{m-d}^n(d; \beta)|^2 |S_0^m(0; \alpha)|^2 |\langle m-d| \psi_{\text{in}}\rangle|^2. \quad (9)$$

From an inspection of Eqs. (6) and (8) we see that when the coefficients $S_m^{n+d} S_0^m$ and $S_{m-d}^n S_0^m$, respectively, change sufficiently slowly with m , then the state $|\psi_{\text{out}}\rangle$, Eq. (2), imitates the state $|\psi_{\text{in}}\rangle$, Eq. (1), but with a *shifted Fock-state expansion*, where the shift parameter is just given by the measured photon-number difference d . Obviously, if $d < 0$ then the state $|\psi_{\text{out}}\rangle$ does not contain any information about the Fock-state expansion coefficients $\langle m| \psi_{\text{in}}\rangle$ for $m < |d|$. With regard to teleportation, this means that the method is conditional. Successful teleportation of a quantum state whose Fock-state expansion starts with the vacuum can only be achieved if the number of photons detected in the mode 1 is not smaller than the number of photons detected in the mode 0. This limitation is exactly of the same kind as in the scheme in [10]: the teleportation fidelity tends sharply to zero as the photon-number difference exceeds some (state-dependent) threshold value. Examples of the coefficients $S_m^{n+d} S_0^m$ are plotted in Fig. 2 for $d = 0$.

To complete the teleportation procedure, Bob transforms the state $|\psi_{\text{out}}\rangle$ applying on it photon-number shifting. Thus, the teleported state is

$$|\psi_{\text{tel}}\rangle = \begin{cases} \hat{E}^{\dagger-d} |\psi_{\text{out}}\rangle & \text{if } d < 0, \\ \hat{E}^d |\psi_{\text{out}}\rangle & \text{if } d > 0, \end{cases} \quad (10)$$

where

$$\hat{E} = \sum_n |n\rangle \langle n+1| \quad (11)$$

(i.e., the operator \hat{A} in Fig. 1 is a power of \hat{E} or \hat{E}^\dagger). The teleportation fidelity is then given by

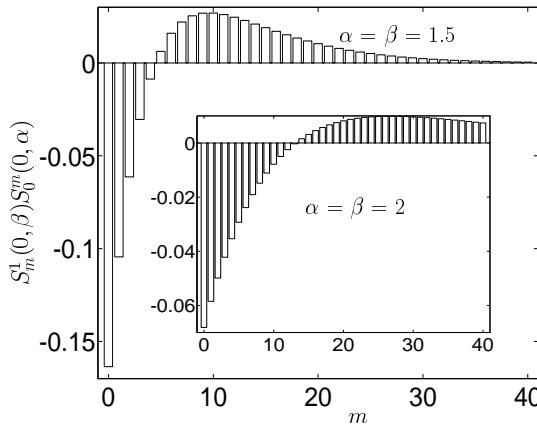


FIG. 2. The product $S_m^{n+d}(d; \beta)S_0^m(0; \alpha)$ is shown for $n=1$ and $d=0$, and the squeezing parameters $\alpha=\beta=1.5$ and $\alpha=\beta=2$.

$$F = |\langle \psi_{\text{in}} | \psi_{\text{tel}} \rangle|^2. \quad (12)$$

For $d \neq 0$, the teleportation scheme requires a realization of the transformations \hat{E} and \hat{E}^\dagger . Unfortunately, there has been no exact implementation of these transformations in quantum optics so far. Photon adding and subtracting are transformations that are very close to the required ones. They are based on conditional measurement and could be realized using presently available experimental techniques [11]. Their use of course reduces the efficiency of the scheme. Thus, the scheme may be presently confined to the case where $d=0$.

III. RESULTS

From Eqs (6) and (8) together with Eqs. (10) – (12), the main results can be summarized as follows. (i) Fock states can perfectly be teleported, i.e., the fidelity, Eq. (12), is equal to unity, which follows from the fact that parametric amplifiers conserve the photon-number difference. Therefore, high teleportation fidelities can also be expected for states with small photon number dispersion. For such states our method may be more suitable than the method in [4], where teleportation via measurement of conjugate quadrature components is realized. On the other hand, high teleportation fidelities are not expected for states with large mean photon number and large photon-number dispersion. In particular, for teleportation of highly excited coherent states or phase squeezed states the method in [4] may be more suitable. (ii) In comparison to the method in [10], our scheme does not require phase shifting of the output state $|\psi_{\text{out}}\rangle$. The squeezing parameters α and β can be chosen such that the coefficients $S_m^{n+d}S_0^m$ and $S_{m-d}^nS_0^m$ in Eqs. (6) and (8),

respectively, are real, so that the Fock-state expansion coefficients $\langle m | \psi_{\text{out}} \rangle$ have the same phase as the coefficients $\langle m-d | \psi_{\text{in}} \rangle$. (iii) A high teleportation fidelity can be expected, provided that the values of the coefficients $S_m^{n+d}S_0^m$ and $S_{m-d}^nS_0^m$ vary sufficiently slowly with m in the relevant range of the Fock-state expansion of the input state $|\psi_{\text{in}}\rangle$. On the other hand, in ranges where the coefficients change rapidly, reliable teleportation cannot be achieved. From Fig. 2 it is seen that the m -range in which $S_{m-d}^nS_0^m$ slowly varies with m increases with the strength of squeezing, and thus the class of states that can be teleported reliably extends.

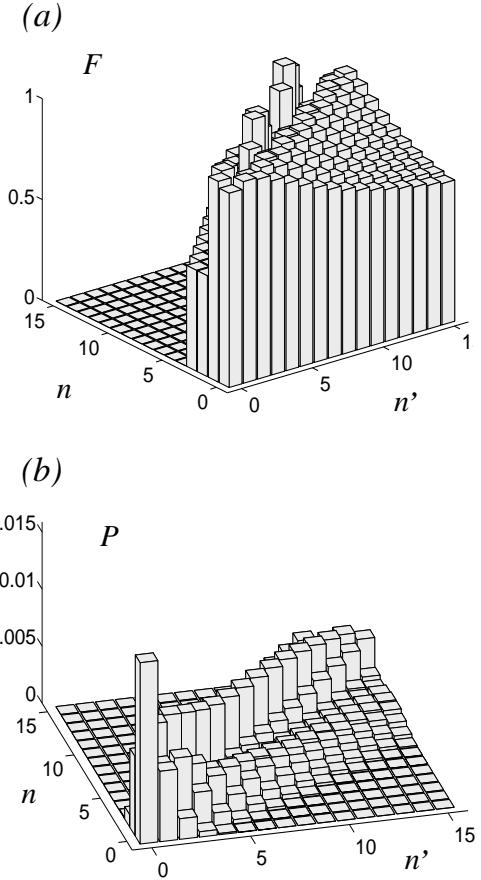


FIG. 3. Teleportation fidelity F (a) and success probability P (b) in dependence on the measured photon numbers n and n' . The input state is $2^{-1/2}(|1\rangle + i|3\rangle)$ and the squeezing parameters are $\alpha=\beta=1.5$.

In order to illustrate the method, we have calculated the teleported state, assuming that input state is a superposition of two Fock states, $|\psi_{\text{in}}\rangle = 2^{-1/2}(|1\rangle + i|3\rangle)$ and equal squeezing parameters α and β are used. Figure 3 presents the dependence on the detected photon numbers n, n' of the teleportation fidelity F , Eq. (12), and the success probability P , Eqs. (7) and (9). We observe that close to the diagonal (but not always directly on it) the fidelity reaches high values close to unity. For the

values of n, n' with $n=n'+2$ and $n=n'+3$ the fidelity is exactly 0.5, which indicates that the Fock state $|3\rangle$ was in the input of the second squeezer and has therefore been re-created in the teleportation. For the values of n, n' with $n > n'+3$ the fidelity drops to zero and so does the probability: such events do not occur for the input state under consideration.

To quantify the performance of the method, we have calculated the probability of events which yield teleportation fidelities larger than or equal to some upper value F_u ,

$$P_u = \sum_{\substack{n, n' \\ F(n, n') \geq F_u}} P(n, n'), \quad (13)$$

where $P(n, n')$ is the success probability of detecting n and n' photons [Eqs. (7) and (9)], and $F(n, n')$ is the corresponding teleportation fidelity. In the example, we find that $P_u \approx 33\%$ for $F_u = 90\%$. Measuring (in place of n and n' in our scheme) the quadrature components X_0 and P_1 in the scheme in [4] would yield (for the same input state and the same squeezing parameter of the entangled state) $P_u \approx 23\%$.

Let us consider the more realistic case where $n = n'$, so that no photon-number shifting is necessary. From Fig. 4 we see that with increasing strength of squeezing a higher fidelity can be realized. However, the corresponding success probability decreases. In the figure, the overall probability of realizing a fidelity higher than 90% is $P \approx 1.97\%$ for the squeezing parameters $\alpha = \beta = 1.5$, whereas for $\alpha = \beta = 2$ the probability reduces to $P \approx 1.06\%$.

Clearly, if the input states are completely unknown, it cannot be predicted with what fidelity a state would be teleported. The scheme applies if the input states can be confined to a certain class of states, so that from an estimated fidelity it can be decided which photodetection results would represent a successful teleportation.

IV. SUMMARY AND CONCLUSIONS

We have suggested a viable modification of the teleportation scheme proposed in [10]. Our scheme avoids the phase sum measurement that is not realizable at present. It uses instead the property of a nondegenerate parametric amplifier that the photon-number difference of the output beams is equal to that of the input beams. However, the price of avoiding phase measurements is a relatively low success probability of teleportation.

Our method and the method in [4], which is based on quadrature-component measurements, may complement one another. So, our method is better suited to teleportation of states with small photon number dispersion (Fock states can be teleported with fidelity equal to unity in principle). The method in [4] is more suitable for teleportation of states with smooth quadrature-component distributions.

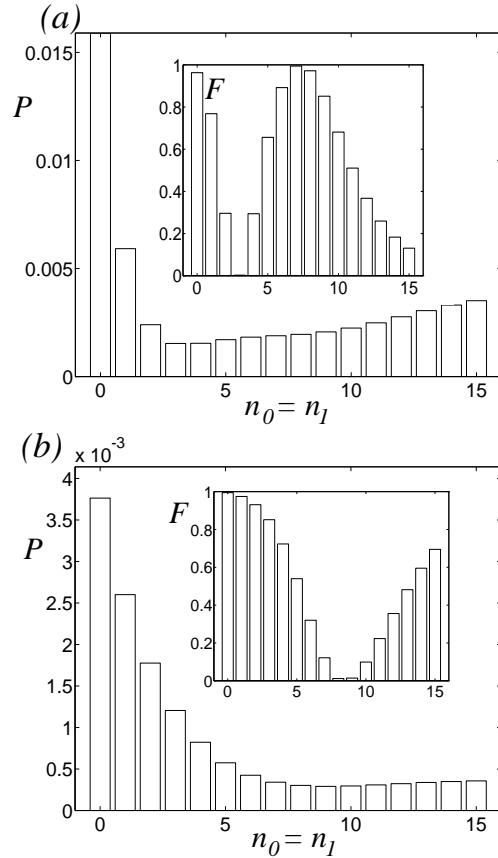


FIG. 4. Teleportation fidelity F and success probability P in dependence on the measured photon numbers $n = n'$. The input state is $2^{-1/2}(|1\rangle + i|3\rangle)$ and the squeezing parameters are $\alpha = \beta = 1.5$ (a), and $\alpha = \beta = 2$ (b).

Although our method is realizable in principle, there are several non-trivial experimental challenges. First, precise photodetection is needed, i.e., detectors are required that are able to distinguish between different photon numbers. This does not only concern Alice's measurement but also Bob's photon-number shifting, e.g., by means of photon adding and subtracting. Second, the photodetection should be sufficiently mode-selective, i.e, one must be able to distinguish whether an incident photon comes from the mode under study or from another part of the spectrum generated by the parametric amplifiers. A central problem in any scheme that exploits quantum coherence is that of decoherence due to unavoidable losses. The effect of decoherence may be reduced, if the squeezing strengths are reduced. However, using smaller squeezing decreases the available teleportation fidelity, so that one has to find an optimum regime for the teleportation of a given class of states, the losses in the scheme, and the required fidelity.

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- [1] Ch. Bennett et al., Phys. Rev. Lett. **70**, 1895 (1993).
- [2] D. Bouwmeester, J.-W. Pan, M. Daniel, H. Weinfurter, and A. Zeilinger, Nature **390**, 575 (1997); D. Boschi, S. Branca, F. DeMartini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
- [3] L. Vaidman, Phys. Rev. A **49**, 1473 (1994);
- [4] S.L. Braunstein and H.J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
- [5] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, Science **282**, 706 (1998).
- [6] T.C. Ralph and P.K. Lam, Phys. Rev. Lett. **81**, 5668 (1998); T. Opatrný, G. Kurizki, and D.-G. Welsch, Phys. Rev. A **61**, 032302 (2000); S. L. Braunstein, G. M. D'Ariano, G. J. Milburn, and M. F. Sacchi, Phys. Rev. Lett. **84**, 3486 (2000).
- [7] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
- [8] M.D. Reid and P.D. Drummond, Phys. Rev. Lett. **60**, 2731 (1988); M.D. Reid, Phys. Rev. A **40**, 913 (1989).
- [9] Z.Y. Ou, S.F. Pereira, H.J. Kimble, and K.C. Peng, Phys. Rev. Lett. **68**, 3663 (1992); Z.Y. Ou, S.F. Pereira, and H.J. Kimble, Appl. Phys. B **55**, 265 (1992).
- [10] G.J. Milburn and S.L. Braunstein, Phys. Rev. A **60**, 937 (1999).
- [11] M. Dakna, L. Knöll, and D.-G. Welsch, Euro. Phys. J. D **3**, 295 (1998); M. Dakna, J. Clausen, L. Knöll, and D.-G. Welsch, Phys. Rev. A **59**, 1658 (1999), Phys. Rev. A **60**, 726(E) (1999); J. Clausen, M. Dakna, L. Knöll, and D.-G. Welsch, Acta Phys. Slov. **49**, 653 (1999).